The Physical Origin of the Scattering Polarization of the Na I D-Lines in the Presence of Weak Magnetic Fields

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ABSTRACT

We demonstrate that the atomic alignment of the hyperfine-structure components of the ground level $S_{1/2}$ of Na I and of the upper level $P_{1/2}$ of the D_1 line are practically negligible for magnetic strengths $B > 10\,\mathrm{G}$, and virtually zero for $B \gtrsim 100\,\mathrm{G}$. This occurs independently of the magnetic-field inclination on the stellar surface (in particular, also for vertical fields). Consequently, the characteristic antisymmetric linear-polarization signature of the scattered light in the D_1 line is practically suppressed in the presence of magnetic fields larger than 10 G, regardless of their inclination. Remarkably, we find that the scattering polarization amplitude of the D_2 line increases steadily with the magnetic strength, for vertical fields above 10 G, while the contribution of alignment to the polarization of the D_1 line rapidly decreases. Therefore, we suggest that spectropolarimetric observations of the "quiet" solar chromosphere showing significant linear polarization peaks in both D_1 and D_2 cannot be interpreted in terms of one-component magnetic field models, implying that the magnetic structuring of the solar chromosphere could be substantially more complex than previously thought.

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1. Introduction

In a recent work, one of the authors (Landi Degl'Innocenti 1998) concluded that his explanation in terms of ground-level atomic polarization of the "enigmatic" linear polarization peaks of the Na I D-lines, observed by Stenflo & Keller (1997) in "quiet" regions close to the solar limb, implies that the magnetic field in the lower solar chromosphere must be either isotropically distributed and extremely weak (with $B \lesssim 0.01\,\mathrm{G}$) or, alternatively, practically radially oriented. That investigation was based on a formulation of line scattering polarization that is valid in the absence of magnetic fields. The suggestion that the magnetic field of the lower solar chromosphere cannot be stronger than about 0.01 G unless it is oriented preferentially along the radial direction was based on the sizeable amount of ground-level polarization required to fit the Q/I observations of Stenflo & Keller (1997), and on the assumption that the atomic polarization of the ground-level of Na I must be sensitive to much weaker magnetic fields than the atomic polarization of the upper levels of the D_1 and D_2 lines.

On the whole, Landi Degl'Innocenti's (1998) argument that the observed linear polarization peaks in the cores of the Na I D-lines are due to the presence of ground-level atomic polarization seems very convincing. However, for a rigorous interpretation of spectropolarimetric observations (e.g., Martínez Pillet et al. 2001; Stenflo et al. 2001) it is of fundamental importance to clarify the physical origin of this polarization by carefully investigating how it is actually produced, and modified by the action of a magnetic field of given strength and inclination.[3]

2. Formulation of the problem

In this Letter we shall focus on the "solar prominence case", in which a slab of solar chromospheric plasma at 6000 K, situated at 10 arcsec ($\approx 7000\,\mathrm{km}$) above the visible solar limb, and permeated by a magnetic field of given strength and orientation, is illuminated

[3]Remarkably, some useful information can be found in the atomic physics literature, notably in the paper by Ellett & Heydenburg (1934) regarding their determination of hyperfine separation constants, and in the work of Lehmann (1969) concerning the orientation of the diamagnetic ground state of Cadmium by optical pumping.

from below (hence, anisotropically) by the photospheric radiation field, which is assumed to be unpolarized and with rotational symmetry around the solar radial direction through the scattering point. The degree of anisotropy of the incident radiation field is calculated as in Landolfi & Landi Degl'Innocenti (1985, hereafter LL85), using the limb-darkening data for the Na I D-lines from Pierce & Slaughter (1982), and a Gaussian absorption profile with $\Delta\lambda_{\rm D} = 41\,{\rm m\AA}$ (corresponding to $T = 6000\,{\rm K}$). The resulting anisotropy factors for the two lines are $w({\rm D}_1) = 0.126$ and $w({\rm D}_2) = 0.118$, where $w = \sqrt{2}\,\bar{J}^2_0/\bar{J}^0_0$ (with $\bar{J}^K_Q$ the radiation field tensors; see, for example, Trujillo Bueno 2001 and note that $-1/2 \lesssim w \lesssim 1$).

In order to investigate this problem, we have applied the quantum theory of spectral-line polarization in the limit of complete frequency redistribution, and in the collisionless regime, as developed by Landi Degl'Innocenti (1983). The excitation of the atomic system is described by a set of ρ^K_Q elements, which are the irreducible spherical tensors of the atomic density matrix (e.g., the review by Trujillo Bueno 2001). We adopt a three-level model of Na I consisting of the ground level ($3\,^2{\rm S}_{1/2}$), the upper level of the D_1 line ($3\,^2{\rm P}_{1/2}$), and the upper level of the D_2 line ($3\,^2{\rm P}_{3/2}$). We also take into account the hyperfine structure (HFS) of Sodium, due to its nuclear spin with $I = 3/2$.

We describe the Zeeman splittings of the Na I levels most generally in terms of the incomplete Paschen-Back effect. In fact, as shown, e.g., in Fig. 5 of LL85, in the 10–50 G range numerous level crossings occur among the magnetic sublevels of the HFS levels with $F = 1, 2, 3$ of ${\rm P}_{3/2}$, whereas fields with strengths in the kilogauss range are necessary to produce level crossing within the HFS levels with $F = 1, 2$ of ${\rm S}_{1/2}$. Therefore, besides population imbalances and quantum interferences (or coherences) between the magnetic sublevels of each F-level, we must also take into account coherences between magnetic sublevels pertaining to different F-levels within a given J-level. Instead, we neglect coherences between the two J-levels ${\rm P}_{1/2}$ and ${\rm P}_{3/2}$, since they are presumably of secondary importance for the generation of the line-core polarization peaks, given the sizeable energy separation between the upper levels of the D_1 and D_2 lines. Our model atom then implies 384 ${}^{JI}\rho^K_Q(F, F')$ elements (with $K = |F - F'|, \ldots, F + F'$ and $Q = -K, \ldots, K$), which are the unknowns of the linear system representing the statistical equilibrium problem for Na I. In this Letter, these quantities are calculated in a reference frame with the z-axis (i.e., the quantization axis) along the solar radial direction through the scattering point.

In summary, our approach is similar to that of LL85, but with the following fundamental improvement. In the expressions of the Stokes components of the emission vector $(\epsilon_I, \epsilon_Q, \epsilon_U, \epsilon_V)$ we now take fully into account the energy separation of the various HFS components of the D_1 and D_2 lines, along with their Zeeman splittings in the presence of the external magnetic field. This is crucial in order to obtain non-zero linear polarization for the

Na I D_1 line.

3. Polarizability of the Na I levels

We have solved numerically the linear system of 384 equations in the unknowns $\rho^K_Q(F, F')$ mentioned before, for magnetic strengths between 0 and 1000 G, and for various inclinations (ϑ_B) of the magnetic field vector from the solar vertical. The eight $\rho^0_0(F, F)$ elements quantify the populations of the various F-levels, and they produce the dominant contribution to the emergent Stokes-I parameter. The ρ^2_Q elements (the *alignment* components) contribute to the *linear* polarization signals, which we quantify by the Stokes parameters Q and U. The ρ^1_Q elements (the *orientation* components) contribute to the *circular* polarization of the scattered radiation. (We recall that in an aligned atomic system, states of different $|M_F|$ are unequally populated, while the populations in M_F and $-M_F$ are the same. In contrast, an oriented system is characterized by different populations in the M_F and $-M_F$ states. We are dealing here with an atomic system which is both aligned and oriented). The contributions from the longitudinal and transverse Zeeman effects are also accounted for, although they become dominant only for relatively strong fields.

Given that we are interested in understanding the generation of *linear* polarization signals in the presence of weak magnetic fields, we focus here on the *alignment* components. In Fig. 1, for each F-level, we show $\sigma^2_0(F) = \rho^2_0(F, F)/\rho^0_0(F, F)$, which quantifies the fractional *population imbalance* of the level. Since the spectral dependence of the incident radiation field is practically negligible over the frequency intervals encompassing the Zeeman components of each of the two spectral lines (*flat-spectrum approximation*), a necessary condition for inducing atomic alignment by means of an unpolarized radiation field is that the illumination of the atomic system be anisotropic. Moreover, atomic orientation can only be originated through the alignment-to-orientation conversion mechanism discussed by Kemp et al. (1984).

Figure 1 shows the sensitivity of $\sigma^2_0(F)$ to the magnetic field strength and inclination. First of all, we note that the largest values are obtained for the level $P_{3/2}$, which can carry atomic alignment even neglecting HFS. On the contrary, both the lower and upper levels of the D_1 line, with electronic angular momentum $J = 1/2$, can carry atomic alignment only because of HFS, as each of these levels splits into two *polarizable* HFS levels with $F = 1$ and $F = 2$. However, it is found that only the level $P_{3/2}$ can be polarized directly via the anisotropic illumination. The levels of the D_1 line, instead, are directly sensitive only to radiation intensity, but they nonetheless become polarized when the atomic polarization of the level $P_{3/2}$ is transferred to the level $S_{1/2}$ via spontaneous emission in the D_2 line, and then from the level $S_{1/2}$ to the level $P_{1/2}$ via radiative absorption in the D_1 line, in a process known as *repopulation pumping* (e.g., Trujillo Bueno 2001). In fact, this explains one of the various remarkable features of Fig. 1, i.e., the fact that the atomic alignment in the lower and upper levels of the D_1 line are equally sensitive to the magnetic strength, independently of the magnetic field inclination.

For instance, we see that a *non-vertical* magnetic field of the order of 0.01 G is sufficient to produce a serious reduction of the atomic alignment of both the lower and upper levels of the D_1 line. This is due to Hanle depolarization of the $S_{1/2}$ ground level, which occurs when the Larmor frequency corresponding to the magnetic field becomes comparable to the inverse lifetime for radiative absorption of that level. Nonetheless, the alignment of the level $F = 2$ of $S_{1/2}$ is still significant for fields up to 10 G, except for field inclinations close to the Van Vleck angle ($\vartheta_B = 54.73°$).

For non-vertical fields the alignment of the level $P_{3/2}$ is also sensitive to magnetic strengths between 0 and 10 G, but the depolarization takes place rather smoothly. An interesting point to note here is the sizeable feedback of the ground-level polarization on the alignment of the F-levels of $P_{3/2}$, with the exception of the level $F = 1$. (This behavior can be understood analytically via inspection of the corresponding transfer rates). As previously indicated, such a feedback takes place because the upper level of the D_2 line can be repopulated as a result of absorptions from the *polarized* ground level.

The most remarkable feature of Fig. 1 is that, independently of the magnetic field inclination (e.g., even for a purely vertical magnetic field), the atomic alignment of each of the two levels involved in the D_1 line transition is suddenly reduced for magnetic strengths larger than 10 G, and practically vanishes for strengths larger than 100 G. We stress the fact that this depolarization is *not* due to the Hanle effect, as it occurs also for vertical fields. A thorough investigation of this phenomenon shows that the vanishing of atomic alignment in the levels with $J = 1/2$ sets in when the electronic and nuclear angular momenta, $\boldsymbol{J}$ and $\boldsymbol{I}$, are decoupled, for the atom in the excited state $P_{3/2}$. In the case of Na I, this decoupling is reached in the limit of the complete Paschen-Back effect of the level $P_{3/2}$, i.e., for magnetic strengths $B \gtrsim 100\,\mathrm{G}$. In such regime, it is found that the transfer of atomic alignment from the level $P_{3/2}$ to the ground level is inhibited. At the same time, the alignment of the level $F = 2$ of $P_{3/2}$ must vanish as well. The analytical proof of these properties will be given in a forthcoming paper (Casini et al. 2001, in preparation).

It is also of interest to note that the repopulation pumping process works efficiently in Na I thanks to the fact that the HFS of the level $P_{3/2}$ is of the same order of magnitude of its natural width. If the frequency intervals between the HFS levels of $P_{3/2}$ were instead substantially smaller than the natural width of this level, then we would have a negligible

HFS interaction during the lifetime of the level $P_{3/2}$, with the result of a drastic reduction in the efficiency of the repopulation pumping process that polarizes the ground level of Sodium, regardless of the magnetic field strength.

4. Observable effects of the atomic alignment

The theory of the Hanle effect for a two-level atom devoid of HFS predicts no modication of the emergent linear polarization with increasing strength of a magnetic field oriented parallel to the symmetry axis of the incident radiation (e.g., Landi Degl'Innocenti 1985). For this reason, and given the conclusions of the previous section, it is of great interest to investigate the emergent polarization of the Na I D-lines for 90° scattering events as a function of the strength of a vertical magnetic field. For this case, and choosing the reference direction for positive Stokes Q parallel to the limb, the only non-zero Stokes parameter is Q.

Figure 2 shows profiles of $Q/I_{\rm max}$, where $I_{\rm max}$ indicates the peak intensity of the emission line, for magnetic strengths between 0 and 100 G. The top panels refer to the solution of the statistical equilibrium problem outlined in § 3. The results shown in the two bottom panels, instead, are obtained assuming that the ground level of Na I is totally unpolarized (e.g., by the presence of depolarizing collisions).

The first interesting feature of the D_2 line polarization is the *increase* of the linear polarization degree as the strength of the vertical magnetic field increases beyond 10 G. This is caused by the interferences, $\rho^2_Q(F,F')$ (not shown in Fig. 1), of the HFS levels in the $P_{3/2}$ level. Our calculations for *inclined* fields with $B \lesssim 100\,{\rm G}$ (not given here) show first a decrease and then an increase of the linear polarization, but the maximum polarization amplitude of the D_2 line corresponds to the $B = 0\,{\rm G}$ case. In general, the polarization of both D_1 and D_2 fluctuates significantly with strength and inclination.

We note that the polarization of the D_2 line is practically unaffected by ground-level polarization for fields larger than 10 G. As shown in Fig. 1, for weaker fields the existing ground-level polarization has a significant feedback on the alignment of the $P_{3/2}$ level, which in turn produces a significant but small enhancement of the emergent linear polarization in the D_2-line core, with respect to the case of totally unpolarized ground level (see Fig. 2).

As shown in Fig. 2, for magnetic strengths $B \lesssim 10\,{\rm G}$, the emergent linear polarization of the D_1 line owes its very existence to the presence of ground-level polarization. Note that its Stokes Q profile is *antisymmetric*.[4] In Fig. 2 we can see that a 1 G *vertical* field has

[4]This peculiar shape has been observed by Trujillo Bueno et al. (2001) in "quiet" regions close to the

practically no effect on the emergent linear polarization, while a 10 G *vertical* field reduces the signal by a factor of two, as a result of the significant reduction of the atomic alignment of the HFS levels of the upper level $P_{1/2}$ (see Fig. 1). What we see for stronger fields in the corresponding top panel of Fig. 2 is the result of the combined action of the atomic alignment of the upper level of the D_1 line and of the transverse Zeeman effect. Since the Zeeman splittings for the magnetic strengths of interest are determined by the incomplete Paschen-Back effect, and the Na I D-lines are the combination of transitions between F-levels weighted by different Landé g-factors, the Stokes Q profiles can be asymmetric, even if the ground level is forced to be totally unpolarized (see bottom panels of Fig. 2). Nonetheless, it can be analitically proven that the wavelength integrated Stokes Q parameter of D_1 is always zero (see LL85). It is also of interest to point out that, if the field is horizontal with randomly distributed azimuth, then the Stokes Q amplitude of the D_1 line is reduced by a factor of four for a magnetic strength of 1 G, with respect to the non-magnetic case.

In general, the linear polarization of the D_1 line is dominated by the transverse Zeeman effect for magnetic strengths $B \gtrsim 50\,\mathrm{G}$ (the alignment of the D_1 levels is practically zero for such field intensities). For example, from Fig. 2 we see that the amplitude of the Stokes Q signal in the presence of a *vertical* field with $B \approx 100\,\mathrm{G}$ is comparable to the amplitude produced by scattering processes in the absence of magnetic fields, but the linear polarization signature has a profile characteristic of the transverse Zeeman effect, with a polarization peak at the line core. On the other hand, the Stokes Q signal in the D_2 line for $B \approx 100\,\mathrm{G}$ is still "scattering like," being determined essentially by the contribution of the alignment of the level $P_{3/2}$. In fact, for this line, the signature of the transverse Zeeman effect begins to appear only for fields $B \gtrsim 500\,\mathrm{G}$.

5. Concluding remarks

One of the most interesting results of this investigation is that the atomic polarization of the HFS levels of the $S_{1/2}$ and $P_{1/2}$ states of Na I is practically negligible for $B > 10\,\mathrm{G}$, and virtually vanishes for $B \gtrsim 100\,\mathrm{G}$, even for a purely vertical field. Consequently, the characteristic antisymmetric scattering polarization signature of the D_1 line is practically suppressed in the presence of fields larger than 10 G, regardless of their inclination.

Concerning the observable effects, we find that the scattering polarization amplitude of the D_2 line increases steadily with the magnetic strength, in the case of vertical fields larger than 10 G, whereas the contribution of atomic alignment to the linear polarization

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solar limb, as shown in Fig. 1 of Trujillo Bueno & Manso Sainz (2001). See also Bommier & Molodij (2002).

of the D_1 line rapidly decreases. On the contrary, for vertical fields such that $B \lesssim 10\,\mathrm{G}$ (or, alternatively, for turbulent or canopy-like fields with a predominance of much weaker fields) it is possible to have a non-negligible scattering polarization signal for the D_1 line, but then the maximum D_2 core amplitude corresponds to the $B = 0\,\mathrm{G}$ case. From this we tentatively conclude that spectropolarimetric observations of the "quiet" solar chromosphere showing significant scattering polarization peaks in both the D_1 and D_2 line cores cannot be interpreted in terms of one-component magnetic field models, suggesting that the magnetic structuring of the solar chromosphere could be substantially more complex than previously thought. For instance, in the presence of a topologically complex distribution of "weak" solar magnetic fields, the D_2 line core would respond mainly to the strongest and preferentially radially oriented fields, while the D_1 line to the weakest and more randomly oriented fields. It remains to be seen whether or not this conclusion is validated after we take fully into account radiative transfer effects and the role of *dichroism* (Trujillo Bueno & Landi Degl'Innocenti 1997) on the emergent polarization of the "enigmatic" Na I D-lines.

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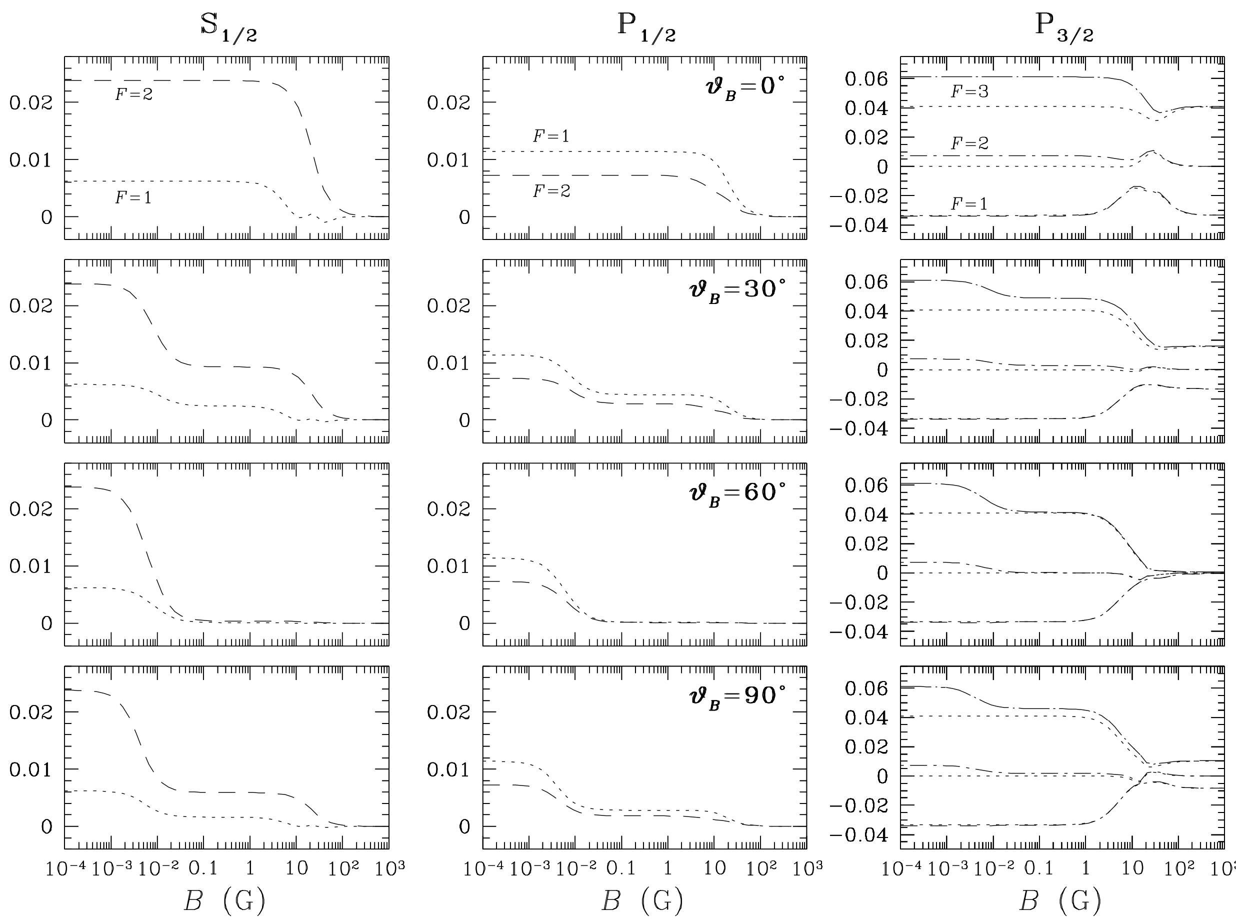

Fig. 1.— The fractional atomic alignment, σ_0^2, in the first three levels of Na I as a function of the magnetic strength and for various inclinations (ϑ_B) of the magnetic field vector. All quantities are referred to a reference frame with the z-axis (i.e., the quantization axis) along the solar vertical through the scattering point. The dotted lines in the panels corresponding to the level $P_{3/2}$ show the σ_0^2 values assuming a completely unpolarized ground level. Note that, even for vertical fields, atomic depolarization of the lower and upper levels of the D_1 line is total when the level $P_{3/2}$ is in the regime of complete Paschen-Back effect.

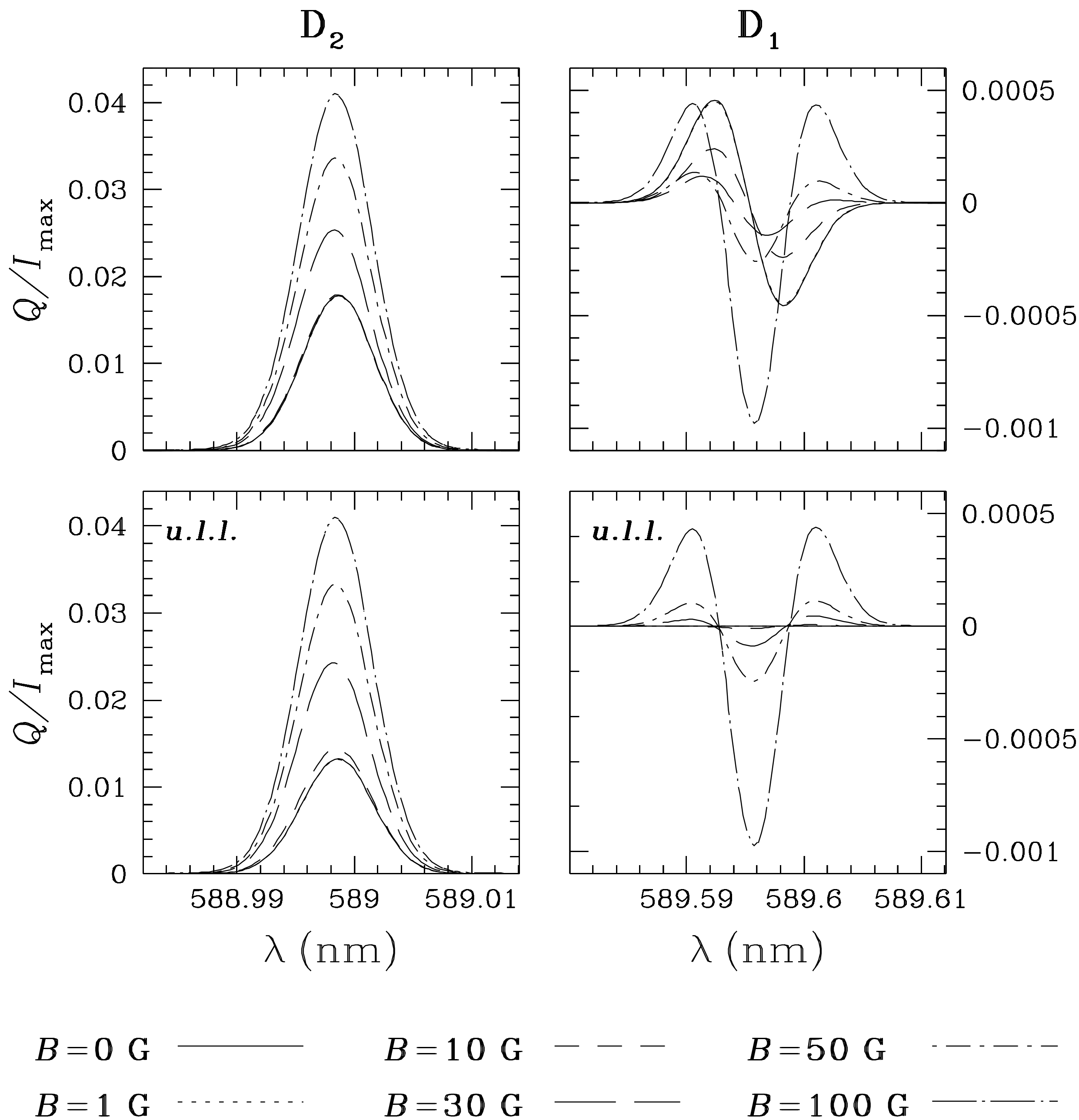

Fig. 2.— The emergent linear polarization of the Na I D-lines in a 90° scattering event for increasing values of the magnetic strength of the assumed vertical field. The positive reference direction for Stokes Q is along the line perpendicular to the radial direction through the scattering point. The top panels take into account the feedback of ground level polarization on the atomic polarization of the two upper levels. The two panels with the label "*u.l.l*", instead, neglect the influence of ground-level polarization. The kinetic temperature is 6000 K. We point out that the amplitude of the D_1 peaks is particularly sensitive to the Doppler width.